

# Efficient generation of NOON states on two microwave-photon resonators\*

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We present an efficient scheme for the generation of NOON states of photons in circuit QED assisted by a superconducting charge qutrit. It is completed with two kinds of manipulations, that is, the resonant operation on the qutrit and the resonator, and the single-qubit operation on the qutrit, and they both are high-fidelity operations. Compared with the one by a superconducting transmon qutrit proposed by Su *et al.* (Sci. Rep. **4**, 3898 (2014)), our scheme does not require to maintain the qutrit in the third excited state with a long time, which relaxes the difficulty of its implementation in experiment. Moreover, the level anharmonicity of a charge qutrit is larger and it is better for us to tune the different transitions of the charge qutrit resonant to the resonator, which makes our scheme faster than others.

**Key words:** entanglement production, NOON states, microwave-photon resonators, superconducting charge qutrit, circuit QED

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## I. INTRODUCTION

Quantum information is an important branch of quantum physics. It includes mainly quantum communication and quantum computation [1–5]. By far, many interesting quantum systems have been presented for quantum information processing, such as nuclear magnetic resonance [6, 7], quantum dots [8–13], diamond nitrogen vacancy (NV) centers [14–19], photonic systems [20–24], circuit quantum electrodynamics (QED) [25–35], and so on. Due to the good scalability [36] and convenient operation on superconducting qubits, circuit QED has attracted much attention in recent years.

Composed of the superconducting circuit and the superconducting 1D resonator, circuit QED [37] has some good characters for completing quantum information processing. The superconducting circuit can act as a qubit perfectly. The energy-level structure of the qubit can be divided into  $\Xi$ ,  $\Lambda$ ,  $V$ , and  $\Delta$  types [38] which can not be found in atom systems. A relative long life time of a superconducting qubit has been realized to reach 0.1 ms [39]. The strong coupling strength between a superconducting qubit and a superconducting resonator [25] has been demonstrated in the experiment. All these characters make circuit QED as a good platform for the quantum computation based on superconducting qubits. In 2009, DiCarlo *et al.* demonstrated a two-qubit algorithms with a superconducting quantum processor [40]. In 2012, Reed *et al.* realized a three-qubit quantum error correction with superconducting circuits [41], and in the same year, Lucero *et al.* computed the prime factors with a Josephson phase qubit quantum processor [36] in which

they integrated five superconducting resonators and four superconducting qubits in a quantum processor.

A superconducting resonator can act as a cavity and a quantum bus, which can be coupled to the distant qubits. The quality of the resonator can be reached to  $10^6$  and even  $10^{12}$  [42], that is, the superconducting resonators can also afford a powerful platform for quantum information processing. In 2007, Schuster *et al.* resolved the photon number states in a superconducting circuit [43]. In 2010, Johnson *et al.* realized a quantum non-demolition detection of single microwave photons in a circuit [44], and in the same year, Strauch *et al.* presented a method to synthesize an arbitrary quantum state of two superconducting resonators [45]. In 2012, Strauch proposed an all-resonant control of superconducting resonators with a drive field [46]. In 2013, we proposed a selective-resonance scheme to perform a fast quantum entangling operation for quantum logic gates on superconducting qubits [47], assisted by one or two superconducting resonators. By combination of the selective resonance and the tunable period relation between a wanted quantum Rabi oscillation and an unwanted one besides the positive influence from the non-computational third levels of the superconducting qubits, these universal quantum gates are significantly faster than previous proposals and do not require any kind of drive fields.

Recently, the generation of the NOON state [48] on two resonators attracted much more attention. In 2010, Strauch, Jacobs, and Simmonds [45] proposed a scheme for completing the generation of the NOON state on two resonators without using the third non-computational excited energy level. The superconducting qubit was operated with a selective rotation by using a drive field whose amplitude should much smaller than the photon-number-dependent Stark shifts on the qubit. That is, the operation time of the qubit should be extended a little longer. In 2010, Merkel and Wilhelm [49] proposed a

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theoretic scheme for generating NOON states on two resonators by using two superconducting qubits and three superconducting resonators. In 2011, Wang *et al.* [50] demonstrated Merkel-Wilhelm scheme in experiment. In Ref.[46], a novel method was proposed to generate the NOON state on two resonators by using a complicated classical microwave pulse and an all-resonant manipulation. It can get a very high-fidelity NOON state within a much shorter time without using the non-computational excited energy level. In 2013, Su *et al.* [51] proposed an interesting scheme for the generation of the NOON state on two resonators with the resonant operation between the transmon qubit and the superconducting resonator, assisted by the single-qubit rotation. The scheme can be completed with  $2N$  steps, and in the first  $N$  steps, the qubit should be maintained in the third-excited state corresponding to the case that the photon number in each resonator is zero.

In this paper, we proposed a scheme to produce the NOON state on two resonators in a quantum processor composed of two tunable superconducting resonators coupled to a tunable  $\Xi$ -type three-energy-level superconducting qutrit. Our scheme requires two kind of quantum operations. One is the resonant operation on the superconducting qutrit and the resonators. The other is the single-qubit manipulation which can be completed by applying a drive field on the qutrit. Our scheme can be used to produce the NOON state on two resonators effectively in a simple and fast way, compared with Merkel-Wilhelm scheme. Moreover, it does not require us to remain the qutrit in the third-excited state all the time, which relaxes largely the requirements of its implementation in experiment, compared with the previous work in Ref. [51].

## II. GENERATION OF THE NOON STATE ON TWO MICROWAVE-PHOTON SUPERCONDUCTING RESONATORS

Let us consider a quantum system composed of two superconducting resonators coupled to a superconducting qutrit, shown in Fig. 1 (a). The energy-level structure of the qutrit is the  $\Xi$  type, which can be found in a superconducting charge qubit, shown in Fig. 1 (b). In order to construct the NOON state on the two resonators  $r_1$  and  $r_2$ , we exploit the lowest three energy levels of the qutrit, denoted by  $|g\rangle_q$ ,  $|e\rangle_q$ , and  $|a\rangle_q$  with the energy  $E_g < E_e < E_a$ . The Hamiltonian of the system composed of the two resonators and the qutrit is (under the rotating-wave approximation, and we choose  $\hbar = 1$  below)

$$H = \sum_{l=g,e,a} E_l |l\rangle_q \langle l| + \sum_{i=1,2} [\omega^{r_i} a_i^\dagger a_i + g_i^{g,e} (a_i^\dagger \sigma_{g,e}^- + a_i \sigma_{g,e}^+) + g_i^{e,a} (a_i^\dagger \sigma_{e,a}^- + a_i \sigma_{e,a}^+)]. \quad (1)$$

Here,  $\omega^{r_i}$  and  $a_i^\dagger$  are the transition frequency and the creation operator of the resonator  $r_i$ , respectively.  $\sigma_{g,e}^+$

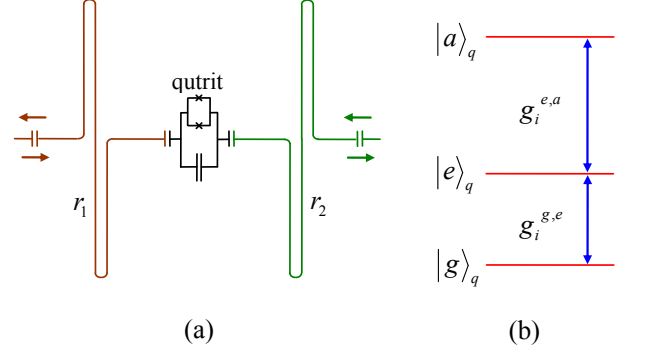


FIG. 1: (a) Schematic diagram for generating the NOON state on two microwave-photon resonator qutrits. There are two resonators coupled to a superconducting qutrit. The transition frequencies of the qutrit and the resonators are tunable. (b) The structure for the energy levels of a charge qutrit.  $r_1$  and  $r_2$  are the two microwave-photon resonators.  $g_i^{g,e}$  ( $g_i^{e,a}$ ) is the coupling strength between the resonator  $r_i$  and the superconducting qutrit in the transition between the states  $|g\rangle_q$  and  $|e\rangle_q$  ( $|e\rangle_q$  and  $|a\rangle_q$ ).

and  $\sigma_{e,a}^+$  are the creation operators of the two transitions  $|g\rangle_q \rightarrow |e\rangle_q$  and  $|e\rangle_q \rightarrow |a\rangle_q$  of the qutrit, respectively.  $g_i^{g,e}$  is the coupling strength between the resonator  $r_i$  and the qutrit in the two transitions  $|g\rangle_q$  and  $|e\rangle_q$ , and  $g_i^{e,a}$  is the coupling strengths between the resonator  $r_i$  and the qutrit in the two transitions  $|e\rangle_q$  and  $|a\rangle_q$ .

In order to turn on or off the interaction between the resonators and the qutrit, on one hand, one can tune the transition frequency of the qutrit by using the external magnetic flux, or tune the transition frequency of the resonator to make them resonate or largely detune with each other. On the other hand, one can tune the coupling strength between the qutrit and the resonator. It worth noticing that a tunable resonator [52] and a tunable coupling qubit [53, 54] have been demonstrated in experiment.

The principle of our scheme for generating the NOON state on two microwave-photon resonators efficiently is shown in Fig. 1(a). Suppose the initial state of the system is

$$\begin{aligned} |\phi\rangle &= \frac{1}{\sqrt{2}}(|g\rangle_q + |e\rangle_q) \otimes |0\rangle_1 |0\rangle_2 \\ &= \frac{1}{\sqrt{2}}(|g\rangle_q |0\rangle_1 |0\rangle_2 + |e\rangle_q |0\rangle_1 |0\rangle_2). \end{aligned} \quad (2)$$

Here the subscripts 1 and 2 represent the two resonators  $r_1$  and  $r_2$ , respectively. That is, the qutrit is in the state  $\frac{1}{\sqrt{2}}(|g\rangle_q + |e\rangle_q)$ , and the resonators are in the state  $|0\rangle_1 |0\rangle_2$ . Here and below,  $|n\rangle_i$  is the Fock state of the resonator  $r_i$ , which means there are  $n$  microwave photons in the resonator  $r_i$  ( $i = 1, 2$ ). To generate the NOON state

[45]

$$|\phi\rangle_{NOON} = \frac{1}{\sqrt{2}}(|N\rangle_1 |0\rangle_2 + |0\rangle_1 |M\rangle_2) \quad (3)$$

on  $r_1$  and  $r_2$  ( $N = M$  is a special situation of the NOON state), our scheme needs  $N + M$  steps. The first  $N$  steps are described as follows.

Step 1: By making both  $r_1$  and  $r_2$  detune largely with the qutrit, one can use a drive field with the frequency equivalent to the transition frequency  $\omega_{e,a}$  of the qutrit to pump the state of the qutrit from  $|e\rangle_q$  to  $|a\rangle_q$ . The amplitude of the drive field is chosen with a proper value for avoiding to pump the state from  $|g\rangle_q$  to  $|e\rangle_q$ . Here  $\omega_{e,a} \equiv E_a - E_e$ . After the operation time  $\Omega_{e,a}t = \pi$  ( $\Omega_{e,a}$  is the proper amplitude of the drive field for pumping the qutrit from  $|e\rangle_q$  to  $|a\rangle_q$ ), the state of the system evolves into

$$\frac{1}{\sqrt{2}}(|g\rangle_q |0\rangle_1 |0\rangle_2 - i|a\rangle_q |0\rangle_1 |0\rangle_2). \quad (4)$$

Subsequently, one can tune the transition frequencies of the qutrit and the two resonators to make  $r_1$  resonate with the qutrit in the transition  $|e\rangle_q \leftrightarrow |a\rangle_q$ . If the coupling strength between  $r_1$  and the qutrit is tuned with a proper value before the resonance, one can neglect the interaction between  $r_1$  and the qutrit in the transition  $|g\rangle_q \leftrightarrow |e\rangle_q$ . Meanwhile,  $r_2$  and the qutrit detune largely with each other. After the interaction time  $g_1^{e,a}t = \pi$ , the state of the system becomes

$$\frac{1}{\sqrt{2}}(|g\rangle_q |0\rangle_1 |0\rangle_2 - |e\rangle_q |1\rangle_1 |0\rangle_2). \quad (5)$$

Step  $j$  ( $j = 2, 3, \dots, N$ ): By repeating the operation of the step 1 for  $N - 1$  times and maintaining  $r_2$  detuning largely with  $r_1$  and the qutrit all the time, the state of the system is changed to be

$$\frac{1}{\sqrt{2}}(|g\rangle_q |0\rangle_1 |0\rangle_2 + (-1)^N |e\rangle_q |N\rangle_1 |0\rangle_2). \quad (6)$$

The whole operation time is

$$t = t_d + t_r. \quad (7)$$

Here,  $t_d = \sum_N \frac{N\pi}{\Omega_{e,a}}$  is the rotated-operation time of the qutrit and  $t_r = \sum_N \frac{\pi}{2g_i^{e,a}\sqrt{N}}$  is the resonated-operation time between the qutrit and the  $r_1$ .

The details of the first  $N$  steps have been described above. The next  $M$  steps are described as follows.

Step 1': By making both  $r_1$  and  $r_2$  detune largely with the qutrit, one can apply a drive field with the frequency equivalent to the transition frequency  $\omega_{g,e}$  of the qutrit to rotate the states of the qutrit with  $|g\rangle_q \leftrightarrow |e\rangle_q$ . By choosing the proper amplitude of the drive field, one can avoid to flip the qutrit with  $|e\rangle_q \leftrightarrow |a\rangle_q$ . After the operation time  $\Omega_{g,e}t = \pi$ , the state of the system evolves from Eq.(6) to

$$\frac{1}{\sqrt{2}}(-i|e\rangle_q |0\rangle_1 |0\rangle_2 - (-1)^N i|g\rangle_q |N\rangle_1 |0\rangle_2). \quad (8)$$

Applying a drive field with the frequency equivalent to the transition frequency  $\omega_{e,a}$  of the qutrit, one can pump the state of the qutrit from  $|e\rangle_q$  to  $|a\rangle_q$ . The amplitude of the drive field is chosen with a proper value for avoiding to pump the state from  $|g\rangle_q$  to  $|e\rangle_q$ . After the operation time  $\Omega_{e,a}t = \pi$ , the state of the system evolves into

$$\frac{1}{\sqrt{2}}(-|a\rangle_q |0\rangle_1 |0\rangle_2 - (-1)^N i|g\rangle_q |N\rangle_1 |0\rangle_2). \quad (9)$$

Subsequently, one can tune the transition frequencies of the qutrit and the two resonators to make  $r_2$  resonate with the qutrit in the transition  $|e\rangle_q \rightarrow |a\rangle_q$ . If the coupling strength between  $r_2$  and the qutrit is tuned with a proper value before the resonance, one can neglect the interaction between  $r_2$  and the qutrit in the transition  $|g\rangle_q \rightarrow |e\rangle_q$ . Meanwhile,  $r_1$  and the qutrit detune largely with each other. After the interaction time  $g_2^{e,a}t = \pi$ , the state of the system becomes

$$\frac{1}{\sqrt{2}}(i|e\rangle_q |0\rangle_1 |1\rangle_2 - (-1)^N i|g\rangle_q |N\rangle_1 |0\rangle_2). \quad (10)$$

Step  $j'$  ( $j' = 2, 3, \dots, M - 1$ ): By repeating the operation of the step 1' for  $M - 2$  times, and maintaining  $r_1$  detuning largely with the qutrit all the time, the state of the system is changed to be

$$\frac{1}{\sqrt{2}}((-1)^{M-1}i|e\rangle_q |0\rangle_1 |M-1\rangle_2 - (-1)^N i|g\rangle_q |N\rangle_1 |0\rangle_2). \quad (11)$$

The final step: Applying a single-qubit operation to complete the rotations of the states  $(-1)^{M-1}i|e\rangle_q \rightarrow i|e\rangle_q$  and  $(-1)^{N-1}i|g\rangle_q \rightarrow |g\rangle_q$ , the state of the system evolves into

$$\frac{1}{\sqrt{2}}(i|e\rangle_q |0\rangle_1 |M-1\rangle_2 + |g\rangle_q |N\rangle_1 |0\rangle_2). \quad (12)$$

By resonating  $r_2$  and the qutrit in the transition  $|g\rangle_q \leftrightarrow |e\rangle_q$ , and making  $r_1$  detune largely with the qutrit, the state shown in Eq.(12) is changed to be

$$\frac{1}{\sqrt{2}}(|N\rangle_1 |0\rangle_2 + |0\rangle_1 |M\rangle_2) \otimes |g\rangle_q. \quad (13)$$

Here, we have generated the NOON state on two microwave-photon resonators efficiently. The operation time of the second  $M$  steps is

$$t' = t'_d + t'_r. \quad (14)$$

$t'_d = \sum_M \frac{M\pi}{\Omega_{e,a}}$  is the rotation-operation time of the qutrit and  $t'_r = \sum_M \frac{\pi}{2g_i^{e,a}\sqrt{M}}$  is the resonance-operation time between the qutrit and  $r_2$ . In which, we neglect the operation time of the single-qubit operation in the final step for generating the NOON state with large number of the  $N$  and  $M$ .

### III. DISCUSSION AND SUMMARY

We have described the process of our scheme for generating the NOON state on two superconducting resonators which are coupled to a  $\Xi$ -type-energy-level structure superconducting charge qutrit. It includes two kinds of quantum operations. The first one is the resonant operation on the qutrit and the resonators. The second one is the single-qubit operation on the qutrit. They are the high-fidelity, high-efficiency, and simple quantum operations in experiment in circuit QED systems. The whole operation time of our scheme for generating the NOON state  $|\phi\rangle_{noon}$  is

$$T = \sum_{j=1}^N \left( \frac{j\pi}{\Omega_{e,a}} + \frac{\pi}{2g_i^{e,a}\sqrt{j}} \right) + \sum_{j'=1}^M \left( \frac{j'\pi}{\Omega_{e,a}} + \frac{\pi}{2g_i^{e,a}\sqrt{j'}} \right). \quad (15)$$

In the calculation for the operation time in our scheme, we neglect the time for changing the transition frequencies of the superconducting qutrit and the superconducting resonator, and the operation time of the single-qubit operation in the final step.

Compared with the one in Ref.[45], our scheme for generating the NOON state on superconducting resonators is much faster as it is composed of the resonant controls. Compared with the one in Refs.[49, 50], both the number of the resonators and that of the qutrits required in our scheme are much smaller as there are three superconducting resonators and two superconducting qutrits in the scheme in Refs.[49, 50], but only two superconducting resonators and a superconducting qutrit used in our scheme. Moreover, the single-qubit operation required in our scheme can be achieved with the simple classical

drive field, and it is simpler than the one used in Ref.[46] as the amplitude of the drive field should be designed with a complex type and it is difficult to be realized in experiment in the latter. In Ref. [51], a similar method is used to generate the NOON state on two resonators. In their work, the transmon qutrit should be maintained in the first  $N$  steps in the third excited state when there is no microwave photons in each resonators. It worth noticing that the higher excited states lead to a lower fidelity operation [46]. Luckily, our scheme does not require us to maintain the qutrit in its third excited state all the time, which relaxes the requirements of its implementation in experiment, compared with the one in Ref.[51]. Compared with a transmon qutrit, the level anharmonicity of a charge qutrit is larger and it is better for us to tune the different transitions of the charge qutrit resonant to the resonator [55].

In summary, we have proposed an efficient scheme to generate the NOON states on two superconducting resonators, assisted by a superconducting qutrit. It requires some high-fidelity quantum operations, that is, the resonant operation on the qutrit and the resonator and the single-qubit operation on the qutrit. Our scheme is a fast and simple one. Moreover, it does not require to maintain the qutrit in the third excited state with a long time, which relaxes the requirements of its implementation in experiment.

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